POLI-Lab @ home

Inclined plane

The project Poli-Lab @ home

The project was created to give the opportunity to establish, even from home, the fundamental contact between what is learned from a theoretical point of view and what happens in nature. In a nutshell, to give students of the Polytechnic of Turin the opportunity, or anyone who has the pleasure and desire, to carry out experiments alongside those proposed at the laboratory. We are, in fact, deeply convinced that where the relationship between theory and natural phenomenon is established, it is possible to understand Physics at a higher level.

Setting up such a project required flexibility and a certain amount of creativity. You can realize this by reading the proposed experiments and realizing them in informal environments ... such as your room.

We would be very pleased if you, students, considered this project as an opportunity. A stimulus to flexibility, critical thinking and creativity. We would be happy if you took seriously the experiments that your teachers will suggest you do, because what they are proposing you is to "play" with the laws of nature. You will have to face and solve small unexpected events, strive to overcome them with ingenuity and creativity. You are part of the training process proposed by the University, so we ask you to give us suggestions and ideas to improve the proposed experiments and to produce a 1 or 2 minute video, which will take you back to work with your experimental apparatus and show us the results that you got. You can send us your ideas and the link to the video by filling out the form on the website of the experiment. Good job!

Activities menu

1. **Acceleration determination with Tracker.**

   [Single student] *Short description.* Yo have to realize a video of an object that slides on an inclined plane and carry on an analysis of the motion with the free software "Tracker". Whit the help of the software Tracker moreover, a parabolic fit of the data $x(t)$ can be made and the acceleration of the body, the friction and their absolute uncertainties are obtained.

2. **Determination of acceleration and dynamic friction coefficient with the least squares method.**

   [Single student] *Short description.* By acting on the data collected with Tracker, we look for the straight line that best fits the experimental data represented in the (t^2, x) plane. The acceleration of the body and the dynamic friction coefficient are obtained from the straight line parameters. The uncertainty of the estimated quantities is evaluated.

List of needed material.

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- An inclined plane (a bench to be tilted, a formica or wooden board, ...)
- 1 body to slide over it
- Smartphone with "bubble level" application to measure the angle of inclination
- Free "Tracker" software (physlets.org/tracker/) compatible with the following operating systems: Windows, OS X, Linux-32 and 64 bit.
- 1 PC with a spreadsheet (Excel, Libre Office, ...).
- 1 extensible meter (sensitivity 1mm).

Acceleration determination with Tracker.

The acceleration of a sliding body on an inclined plane is determined through a video recording of the motion and a video analysis with software Tracker . Once we know the acceleration and the inclination angle of the plane, the friction can be determined.

In the preparation phase of your experimental apparatus, it is good to keep in mind the main degrees of freedom on which you can act to define the operating conditions:

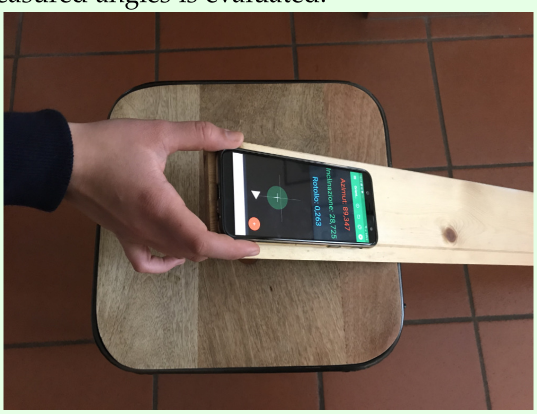
1. The **body mass** will play an important role in determining the friction, light bodies will have less friction (with the same inclination)
2. The textbftilt angle will affect both the component of gravity acceleration along the plane and the friction. By increasing the angle, the component of g along the plane will increase, and the component perpendicular to the plane (and also the friction) will decrease.

Activity 1. How to carry out the experiment.

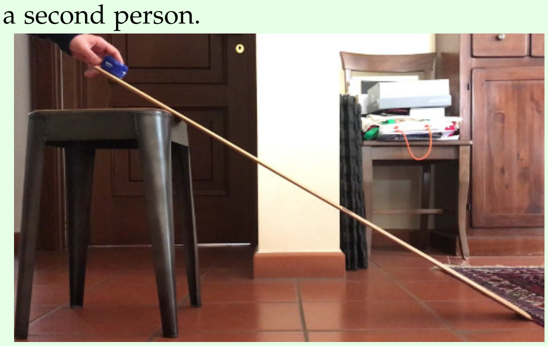
- Create an inclined plane (by tilting a bench, a shelf, an axis, ...).

- When choosing the angle of inclination, keep in mind that an excessively fast motion will make less precise the tracking of the position in the various frames of the video.
- One possible way to determine the inclination is as follows: increase the angle until slightly exceeding the inclination which allows the component of the weight force along the plane to exceed the maximum static friction force. In this way, when the body is released with zero initial speed, it will begin to slide without the need for external intervention.
- Measure the length of the inclined plane which will be the reference d_{ref} length that will allow the analysis software to convert lengths from pixels to meters.
- In choosing the location of the apparatus, if possible, choose a place that allows you to have a uniform colored background, with a color in contrast with those of the body that will slip.

- Stand at the side of the inclined plane and set the video camera of a smartphone or tablet to shoot the movement of the body.
- Place the camera lens at about the midpoint of the body path.
- Care must be taken in order to fit the entire inclined plane in the frame, since its length will be a reference for Tracker. To minimize the relative errors on distances it is also recommended to make sure that the tilted plane covers a high percentage of the size of the frame.
- Before let the body slip along the surface, measure its angle of inclination. For example with a "bubble level" application. A mobile phone can be placed along the inclined plane with the two opposite orientations and the average of the measured angles is evaluated.



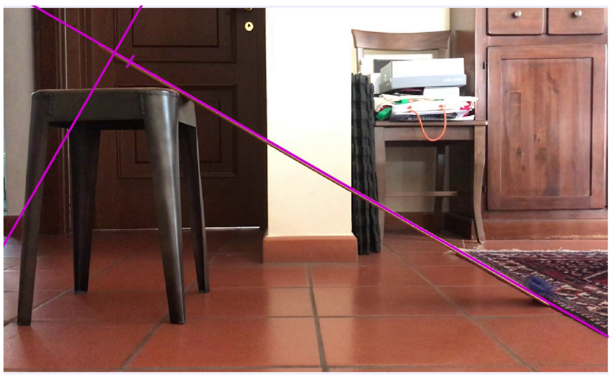
- Alternatively, the tilt angle can be measured using trigonometric formulas for right triangles measuring the length of the plane and the horizontal projection.
- Place the body at the top of the inclined plane and leave it free to slide downwards from a standing position. The descent of the body is filmed. You will need to get help from a second person.



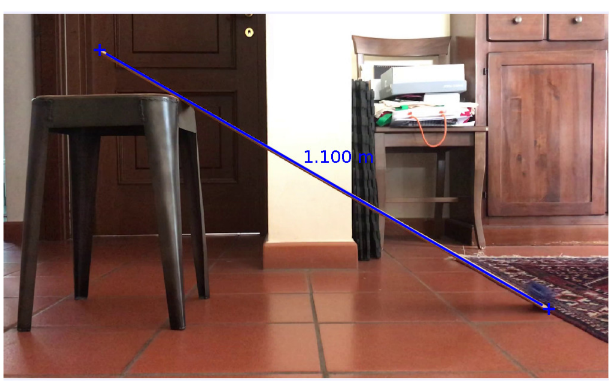
Activity 1. Analysis with Tracker

For a step-by-step guide of the analysis, you can consult VIDEO5 in which a real analysis is commented and taken up again. Here we present the general lines of the procedure.

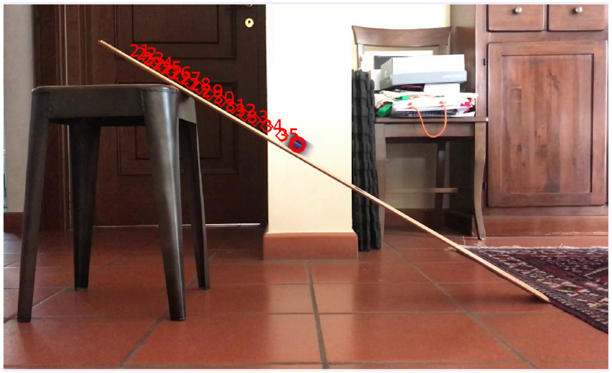
- To allow the software to determine the (x,y) position of the body, a Cartesian reference system is defined. It is oriented so that the x-axis is directed along the inclined plane and the origin is placed in the initial position of the body.



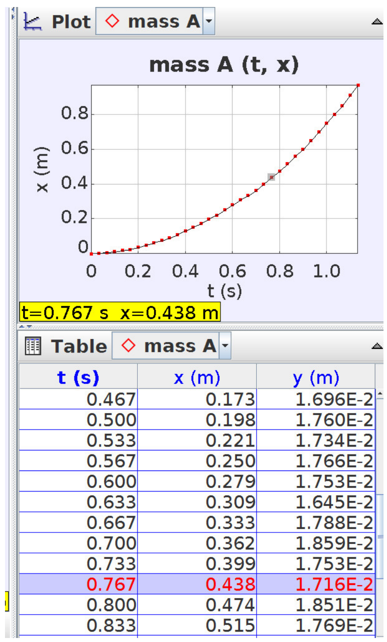
- Define the reference length d_{ref} that allows the software to convert measured distances in units of pixels into meters.



- Trace the position of the body in the frames of the descent.



- As body position is tracked, Tracker offers both numeric data and the graph of the x(t) function.



t (s)	x (m)	y (m)
0.467	0.173	1.696E-2
0.500	0.198	1.760E-2
0.533	0.221	1.734E-2
0.567	0.250	1.766E-2
0.600	0.279	1.753E-2
0.633	0.309	1.645E-2
0.667	0.333	1.788E-2
0.700	0.362	1.859E-2
0.733	0.399	1.753E-2
0.767	0.438	1.716E-2
0.800	0.474	1.851E-2
0.833	0.515	1.769E-2

- You can determine the acceleration of the body motion by trying to fit the experimental data with a parabolic function of the type

$$x(t) = At^2 + Bt + C \tag{1}$$

.

- Open the analysis window offered by Tracker (View / Data Tool (Analyze)) and the fit between parabola and experimental points can be made. This operation is carried out by varying the values of parameters A, B, C with "clickable" arrows that also allow you to make very fine variations. In real time, the user sees the parabola change according to the changes introduced and adhere better to the experimental points.

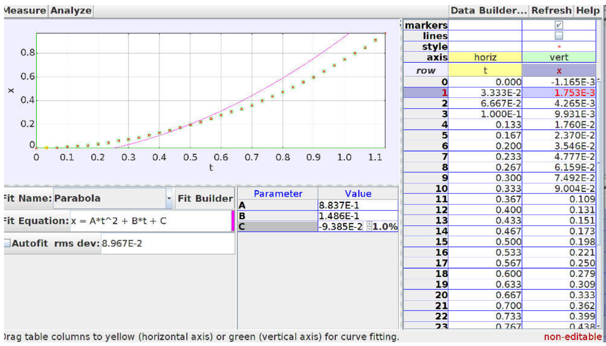


Figure 1: Adaptation of the parabola to the experimental points.

During the fit operation it will be necessary to keep in mind that the sliding body starts from a position $x_0 = 0$ and still. Therefore parameters B and C can be set directly to zero. In other words, the parabola with which we will try to model the experimental data will be of the type:

$$x = At^2 \,. \tag{2}$$

Once the fitting operation is finished, we will have the value of parameter A (whose dimensions are m/s^2) and we can derive the acceleration of the body from it:

$$a = 2A \,. \tag{3}$$

From the value of the tilt angle we can estimate the component of the acceleration of gravity along the inclined plane. Finally, by subtracting the body's acceleration a from the latter, we can derive the acceleration due to friction, oriented in the opposite direction to that of motion:

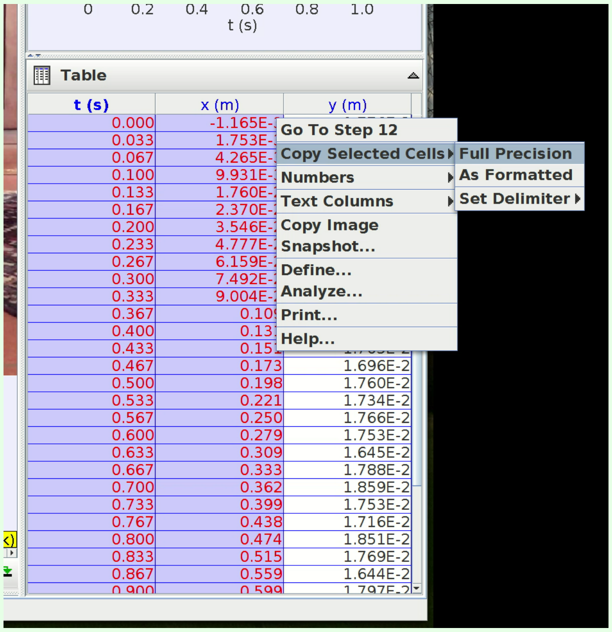
$$a_{fr.} = g \cdot sin\theta - a \,. \tag{4}$$

Activity 1. Determination of uncertainties.

To estimate the uncertainty about acceleration a, we can use the following procedure.

Activity 1. Evaluation of uncertainty on a

- From the main Tracker window, under the graph $x(t)$, select the data columns with time and position. Then by clicking on them with the right button select "Copy Selected Cells" and "Full Precision".



- Open a spreadsheet (Excel, Libre Office, ...) and paste the copied data columns.

- We are assuming that the equation of motion is

$$x = \frac{1}{2}at^2 \tag{5}$$

therefore, evaluating for each data pair (t, x) the quantity $2x/t^2$ we will have numerous estimates of the acceleration of the body during the motion.

- Create in your spreadsheet, next to the column of the x, a new column where you will calculate the value $2x/t^2$ expressed in m/s^2.

- The first data (cells on a green background) are affected by a very large relative uncertainty, as the initial displacement of the body (difference between consecutive x) is in the order of millimeters or a few centimeters.

t (s)	x (m)	2x/t^2 (m/s^2)
0.00E+00	-1.17E-03	
3.33E-02	1.75E-03	3.155171595
6.67E-02	4.27E-03	1.9193988063
1.00E-01	9.93E-03	1.9861006062
1.33E-01	1.76E-02	1.9798256676
1.67E-01	2.37E-02	1.706545831
2.00E-01	3.55E-02	1.772939387
2.33E-01	4.78E-02	1.7549097165
2.67E-01	6.16E-02	1.7322574026
3.00E-01	7.49E-02	1.664988756
3.33E-01	9.00E-02	1.6206335065
3.67E-01	1.09E-01	1.6166933298
4.00E-01	1.31E-01	1.6313535138
4.33E-01	1.51E-01	1.6046188639
4.67E-01	1.73E-01	1.5897189671
5.00E-01	1.98E-01	1.581841928
5.33E-01	2.21E-01	1.5524534379
5.67E-01	2.50E-01	1.558911413
6.00E-01	2.79E-01	1.5500456222
6.33E-01	3.09E-01	1.5396791676
6.67E-01	3.33E-01	1.4983574943
7.00E-01	3.62E-01	1.47853734

- By neglecting these data and considering only the values of a on a blue background, we can evaluate the average value of a which must be close to the value obtained through the fit made with Tracker.

$$\bar{a} = \frac{1}{N}\sum_{i=1}^{N} a_i \tag{6}$$

  N indicates the number of data available, a_i the generic value of the acceleration calculated in the rows of the column $2x/t^2$.

- The uncertainty of a will be the standard deviation of the column of values $2x/t^2$.

$$\delta a = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(a_i - \bar{a})^2}$$

- The acceleration of the body will therefore be expressed by the value obtained with the Tracker fit and by the uncertainty just calculated.

To complete the experience, we can calculate the uncertainty of friction acceleration

$$a_{fr.} = g \cdot sin\theta - a\,. \tag{7}$$

Activity 1. Estimate of the uncertainty of the friction acceleration.

- As can be seen from the relation(7) the uncertainty on the $a_{fr.}$ will be due to the propagation of the error of θ and a (on the website you will find a short VIDEO that resumes the technique of calculating uncertainty through differentiation).

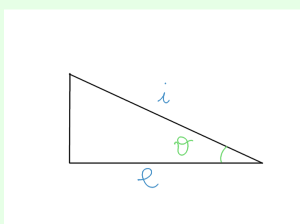
- **Caution!**: remember to use values of angles expressed in radians since the usual derivation rules for goniometric functions are such only if the angles are expressed in radians.

- To determine the uncertainty on θ we can follow two ways depending on how the angle has been measured. **1.** If the angle is the average of two measurements with the smartphone, the semi-dispersion of the 2 measures can be considered as uncertainty:

$$\delta\theta = \frac{|\theta_1 - \theta_2|}{2}. \tag{8}$$

2. If the angle has been established starting from the measurement of the sides of a right triangle, the error of the measurements of the sides on the angle must be propagated. For example if the angle was measured by

$$l = i\, cos(\theta) \tag{9}$$



You can differentiate the (9) and derive $\delta\theta$ from δl and δi, which are known. They will be in the first approximation equal to the instrumental error.

- At the end of these calculations the friction acceleration will be expressed as

$$a_{attrito} = (g \cdot sin\theta - a) \pm \delta a_{attr.}\,. \tag{10}$$

Activity 2. Determination of acceleration with the least-squares method.

To carry out this activity it is necessary to previously perform the body tracking operation with Tracker (see section **Activity 1. Analysis with Tracker**). The starting point consists of the time and position data that appear in the columns at the bottom right of the usual Tracker main screen.

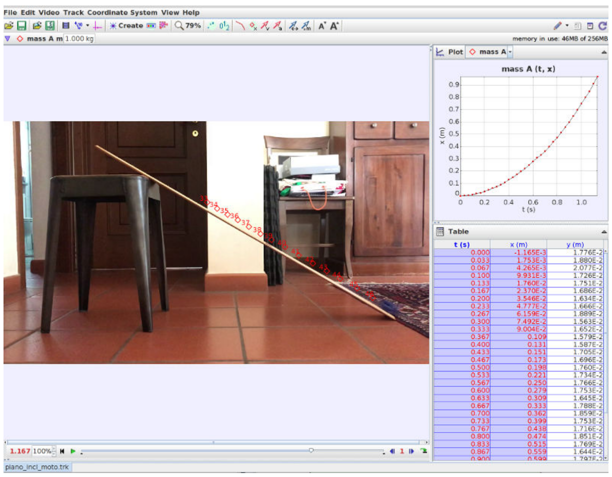


Figure 2: The main screen of Tracker, as it appears after the operation of tracking the material point associated with the body. Bottom right: data columns (time, position).

With the least squares method we will determine the parameters of the equation of the straight line which expresses the relationship between the position x and time squared t^2.

In practice, we have an expected theoretical trend

$$x = \frac{1}{2}at^2 \tag{11}$$

that we want to make linear and then apply the least-squares method. In order to treat the quadratic relation (11) as a linear relation it is necessary to establish the correspondences shown in the figure (3) and consider the position as y, the time squared as x and the coefficient of t^2 as slope:

$$x \to y, \quad \frac{1}{2}a \to c, \quad t^2 \to x\,. \tag{12}$$

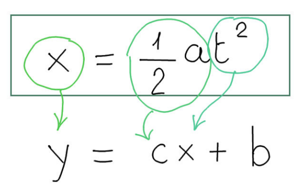


Figure 3: Scheme of the correspondences between the physical variables and those typical of the least-squares method.

In this way the expected theoretical relationship becomes of the type

$$y = cx + b \tag{13}$$

and with the least squares method we will try to determine the parameters c and b that best fit the experimental data. Be careful not to get confused, to the right of these correspondences are the physical quantities of our inclined plane (position, (acceleration) / 2 and (time)

^{2}) on the left the quantities that we will use to apply the least-squares method. Once the parameters c and b have been determined with their uncertainties, we can proceed to evaluate the acceleration of the body and the uncertainty about acceleration.

We can interpret what we are about to do also from a geometric point of view. We are going to graphically represent our data on a Cartesian plane (t^2, x).

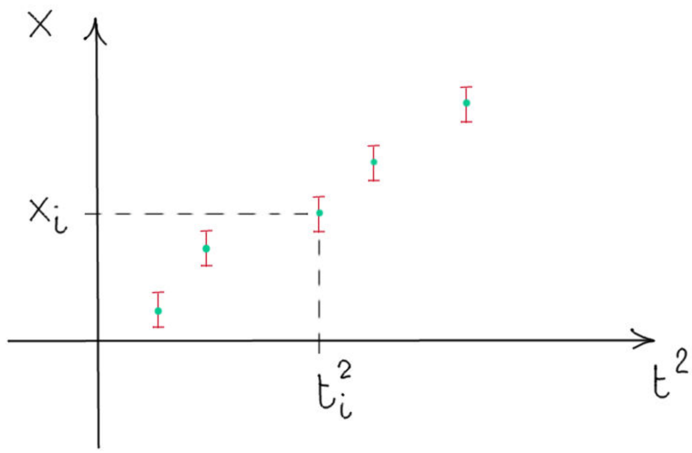


Figure 4: Graphical representation of data on a plane (t^2, x).

With the least squares method we will search for the straight line that best fits the data and we will determine the value of the slope c and the intercept b (which we expect close to zero).

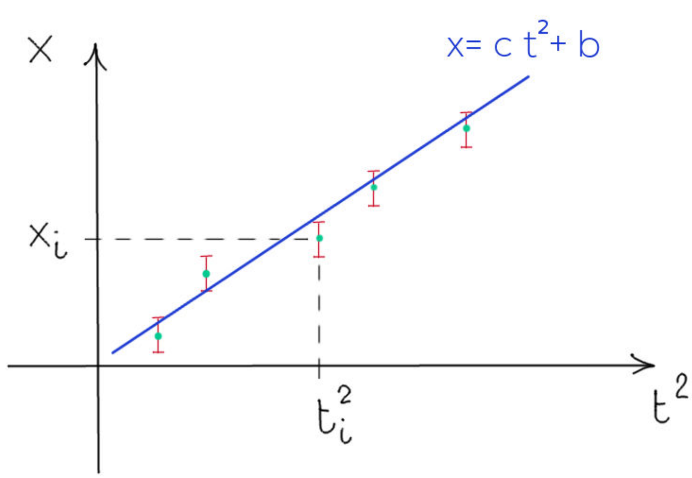


Figure 5: With the least squares method we find the straight line equation that best fits our experimental data.

Activity 2. The least squares method: calculation of the parameters c and b.

- The goal is the determination of the straight line that best fits our experimental data (t_i^2, x_i). In particular, find the values of c and b that define this straight line.
- The subscript i ranges from 1 to N, where N represents the number of data collected with Tracker.
- From here on we will indicate the values of the independent variable t_i^2 with the symbol x_i, in fact it represents the abscissas of our data. The dependent variable x_i will be indicated by the symbol y_i.
- Theory tells us that to determine the 2 parameters c and b, it is necessary to calculate 4 quantities:

$$\sum_{i=1}^{N} x_i \quad , \sum_{i=1}^{N} y_i \quad , \sum_{i=1}^{N} x_i^2 \quad , \sum_{i=1}^{N} x_i y_i$$

- A spreadsheet (Excel, Libre Office, ...) can help us in reaching our goal. First we can define two columns of data. One for x_i, the other for y_i.
- Now let's fill 2 new columns. In one, alongside the data columns, we will calculate x_i^2, in the next the product $x_i\, y_i$.
- At this point we should have 4 columns of numeric values. In the first we will have the values of x_i, in the second the values y_i, in the third the x_i^2, in the fourth the $x_i\, y_i$.
- In order not to get confused we leave an empty row under these 4 columns and in the next row we calculate the sums of all the above elements. So, for example, under the x_i column we will evaluate the $\sum_{i=1}^{N} x_i$.

- Now it remains to calculate the values of the parameters c and b of the line that will represent the linear link between our variables. We will have to use these formulas (all summations are to be understood with i ranging from 1 to N):

$$\Delta = N \sum x_i^2 - \left(\sum x_i\right)^2 \tag{14}$$

$$c = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta} \tag{15}$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta} \tag{16}$$

At this point we should have found the parameters c and b that define the straight line that best fits our data. We can represent it in superposition to the graph of the experimental points. In this way we will obtain a graph similar to that of the figure (5). Analyzing it we will be able to immediately make two rough checks on the calculations carried out so far:

1. The line should adapt to our experimental points. If it passes away from them or shows a slope in striking disagreement with what the experimental data allow us to glimpse, probably some calculations will have got out of hand.

2. The theoretical model tells us that the linear relationship between x and t^2 is expressed by a straight line passing through the origin. We therefore expect a small value for the parameter b. Another alarm bell, then, will come from any intercept values that are conspicuously different from zero.

Once these checks have been done, we continue with the analysis. Now it is essential to calculate the uncertainty with which we have estimated the slope c and the intercept b of the line. The reason, for example, is that we will use the value of c to estimate the acceleration a of the body, we will therefore need the uncertainty of c and then propagate it on the uncertainty of the acceleration.

Activity 2. The least squares method: the uncertainty of c and b.

- The uncertainties on the parameters are assessed with these formulas:

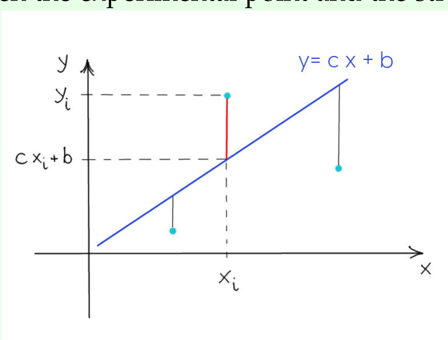
$$\delta c = \delta y \sqrt{\frac{N}{\Delta}} \quad , \quad \delta b = \delta y \sqrt{\frac{1}{\Delta} \sum x_i^2} \tag{17}$$

  where δy represents the uncertainty on the experimental ordinates, that is, on the various positions x_i assumed by the body during its descent motion. To understand, those are indicated by the red error bars in the figure (4).

- The uncertainty δy can be calculated using this relationship

$$\delta y = \sqrt{\frac{1}{N-2} \sum_{i=1}^{N} (c x_i + b - y_i)^2}\,, \tag{18}$$

  which expresses a sort of mean value of the quadratic difference between the straight line found and the experimental points. Looking at the figure below, in fact, it can be seen that the difference in the sum of the argument is, in modulus, the length of the red segment: the ordinate distance between the experimental point and the straight line.



Having found the uncertainties on the parameters of the line that best fits the experimental points, we can refine our control over the results of the analysis. As mentioned above, we expect b to have a value close to zero. Now we are able to verify if the variability range of the parameter b, that is the interval $[b - \delta b, b + \delta b]$ includes zero. That said, let's move on to the final phase, that is to estimate the value of a starting from the slope c of the line.

Activity 3. Calculation of a and its uncertainty.

As already mentioned several times, from theory we know that the relationship between x and t^2 is expressed by the equation

$$x = \frac{1}{2}at^2\,. \tag{19}$$

Now, however, we have an estimate of the slope of this relationship: the parameter c. We can therefore write

$$c = \frac{1}{2}a \tag{20}$$

and from this equation we can derive the value of a.
Let's proceed:

- We calculate a using the value of the parameter found with least-squares:
$$a = 2c. \tag{21}$$
- Using the uncertainty of c (17) we evaluate the uncertainty δa on the value of a. For this purpose, the usual calculation technique is used for the propagation of the error.
- We conclude by expressing the numerical value of a obtained with this procedure:
$$a = 2c \pm \delta a. \tag{22}$$

Activity 2. Determination of the friction acceleration and the friction coefficient.

We can exploit the results obtained with the least squares method to calculate the friction acceleration and the dynamic friction coefficient.

Acceleration due to friction is given by expression

$$a_{fr.} = g \cdot sin\theta - a\,, \tag{23}$$

that we can evaluate using the value of a obtained with least squares, the value of θ measured and assigning g the measured value, for example, with the experience of the pendulum. The uncertainty about $a_{fr.}$ Will be estimated by propagating the errors of g, θ and a (on the site page you will find a short VIDEO which takes up the technique of calculating uncertainty through differentiation). As always, textbf caution, when calculating the propagation of uncertainty, to express the angle in radians!

As for the dynamic friction coefficient μ_D, remember that

$$a = g\, sin\theta - \mu_D\, g\, cos\theta \tag{24}$$

By specifying the friction coefficient from (24) we can estimate its value. By propagating the uncertainties of g, θ, a, we reach uncertainty $\delta a_{fr.}$.