POLI-Lab @ home

Simple Pendulum

Project Presentation Poli-Lab @ home

The project was created to give the opportunity to establish, even from home, the fundamental contact between what is learned from a theoretical point of view and what happens in nature. In a nutshell, to give students of the Polytechnic of Turin the opportunity, or anyone who has the pleasure and desire, to carry out experiments alongside those proposed at the laboratory. We are, in fact, deeply convinced that where the relationship between theory and natural phenomenon is established, it is possible to understand Physics at a higher level.

Setting up such a project required flexibility and a certain amount of creativity. You can realize this by reading the proposed experiments and realizing them in informal environments ... such as your room.

We would be very pleased if you, students, considered this project as an opportunity. A stimulus to flexibility, critical thinking and creativity. We would be happy if you took seriously the experiments that your teachers will suggest you do, because what they are proposing you is to "play" with the laws of nature. You will have to face and solve small unexpected events, strive to overcome them with ingenuity and creativity. You are part of the training process proposed by the University, so we ask you to give us suggestions and ideas to improve the proposed experiments and to produce a 1 or 2 minute video, which will take you back to work with your experimental apparatus and show us the results that you got. You can send us your ideas and the link to the video by filling out the form on the website of the experiment. Good job!

Activities menu

1. **Measurement of the pendulum period and the acceleration of gravity with Tracker.**

   [Single student] *Short description*. Construction of the pendulum, creation of a video of the oscillations, analysis of the video with software Tracker, determination of the period through the sinusoidal fit of the diagram x(t).

2. **Repeated measures for determining the period of a pendulum and the acceleration of gravity.**

   [Single student] *Short description*. Pendulum construction, repeated measurement of the period with a stopwatch, determination of the average value and uncertainty on each individual measurement, calculation of the uncertainty on the average value, evaluation of the acceleration of gravity and its absolute uncertainty.

3. **Study of the dependence of a pendulum's period on its length and determination of the acceleration of gravity.**

   [Collaboration between at least 5 students] *Short description*. Pendulum construction, each component of the collaboration determines the period of a pendulum with different length, linear fit with the least squares method of the relationship between the squared period and length, determination of the acceleration of gravity and its absolute uncertainty starting from the parameters of the fit.

List of needed material and construction of the pendulum

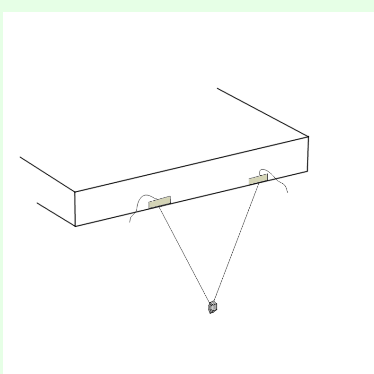
List of needed material

- Thin sewing thread.
- 1 light mechanical nut or a metal washer (we recommend a mass between 5 and 15 grams).
- Adhesive tape (preferably paper tape).
- (For activity n ° 1) Smartphone or Tablet with video camera.
- (For activity n ° 1) Free "Tracker" software (physlets.org/tracker/) compatible with the following operating systems: Windows, OS X, Linux-32 and 64 bit.
- 1 PC with spreadsheet software (Excel, Libre Office, ...).
- 1 extensible meter (precision 1mm).

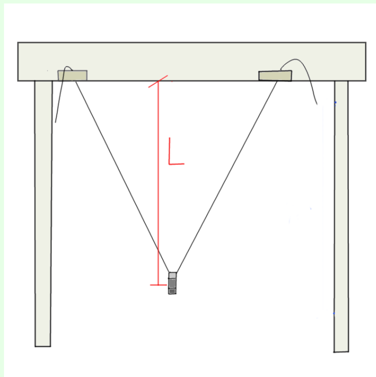
Pendulum construction.

The pendulum is made with thin sewing thread and a light mechanical nut (we recommend mass between 5 and 15 grams), the ends of the thread are fixed with adhesive tape (paper if possible) to the edge of a table or a desk. It is also possible to think of fixing the pendulum to the crossbar of a door.

- *As shown in the figure, the adhesive tape must be aligned and "flush" with the lower edge of the support to which the pendulum is fixed (desk edge, table, beam). This is to ensure that the two ends of the wire can rotate freely around the contact points.*



- **Caution!**. *The pendulum length L will be the distance between the center of gravity of the suspended mass (the center of the nut) and the lower edge of the support.*



- *The oscillating mass will naturally move into a symmetrical position with respect to the two suspension points. However, we recommend a quick visual check before using the pendulum for measuring activities.*

General precautions and approximations.

- *Build the pendulum so that it is as long as possible (from 40 cm upwards).*

When conducting an experiment it is essential to pay attention to the *minimization of relative errors*. It is therefore always advisable to make sure that the values of the quantities we will measure are large with respect to the errors we will make. In our case, for example, having to measure oscillation times and pendulum length, it will be convenient for us to build a pendulum of long length (compatibly with the limitations of the spaces). The greater the length (say from 40 cm upwards), the lower the relative error on the length and the period (the oscillations will be slower, therefore the error introduced by our reaction times will be less important).

- *Limit to oscillation amplitudes that are less than 1/10 of the pendulum length L.*

  In the Physics 1 course the simple pendulum is modeled in the approximation of small oscillations ($sin\theta \sim 0$), a choice that allows us to consider the motion of the oscillating mass as harmonic and one-dimensional.

Activity 1. Period and gravity acceleration measurement with Tracker.

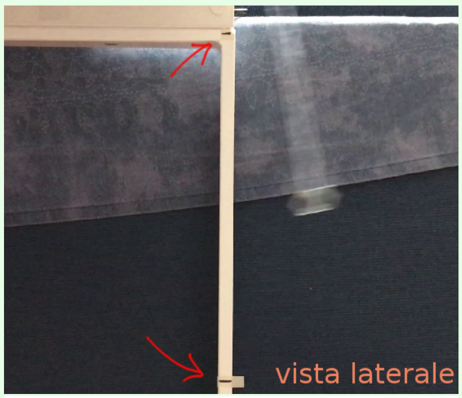
The purpose of this activity is to measure the oscillation period of a pendulum starting from the analysis of a video of the oscillations of the built pendulum. The analysis is carried out in the framework of the Tracker software which allows to "track" the position of the oscillating mass during its motion. After determining the period of the pendulum, the acceleration of gravity will be assessed from the relationship that binds it to the period and length of the pendulum. Finally we will evaluate the absolute uncertainty of the acceleration of gravity by calculating the error propagation.

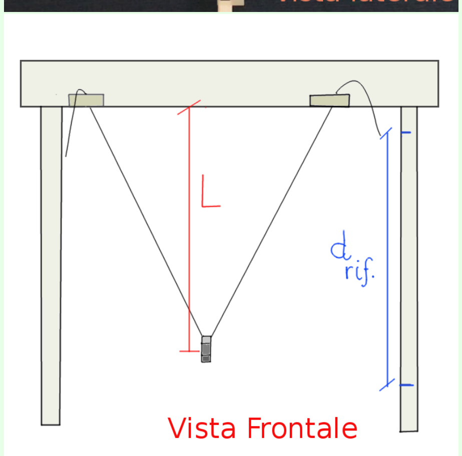
Let's start from the situation in which the pendulum has already been made following the indications of the section dedicated to the construction of the apparatus. The first goal to be achieved, therefore, is to make a movie that takes a certain number of pendulum oscillations.

Activity 1. Motion video. Operating Procedure.

- In order to be able to trace the oscillating mass with Tracker software it is better that its image to be clearly distinguishable from the background (example: light mass, dark background or vice versa).
- Measure and note the pendulum length L.
- Establish and write down the uncertainty about L. We expect it to be greater than the sensitivity of the measuring instrument used (the extensible meter has a sensitivity of 1mm). Therefore, to avoid underestimation, we can assign the value of 5 mm to the uncertainty δL.

- Place in the background, in the immediate vicinity of the pendulum (e.g. on one table leg), two pieces of adhesive tape (see figures) on which draw 2 clearly visible notches. Measure and note the distance d_{ref} between the two notches. This will be used to inform the software about the values of the distances in the image. To minimize the relative error it is good that the d_{ref} occupies a high percentage of the height of the frame of the movie.





- Place yourself in a position where the shooting direction lies on the plane formed by the 2 wires of the pendulum in the rest position. From a practical point of view, it is sufficient to place the mobile phone in a vertical position, on the side of the pendulum, in a position from which the two wires appear superimposed on each other. In this way the shoot is perpendicular to the direction of motion of the oscillating body.

- Shoot, making sure that the two notches that determine the reference distance d_{ref} appear in the frame. Finally, make sure that the amplitude of the swing occupies a good percentage of the frame.

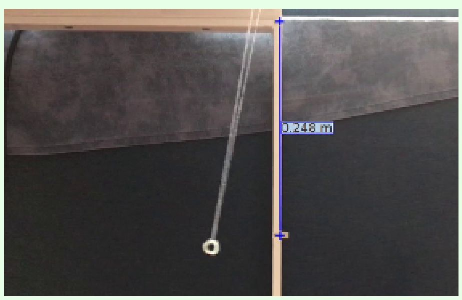
Once the video has been made, it can be imported in the computer and analyzed with the software Tracker.

Activity 1. Analysis with Tracker.

Let's give a general idea of how motion analysis is conducted with Tracker. VIDEO4, associated with this document, shows step by step and dynamically, how to operate.

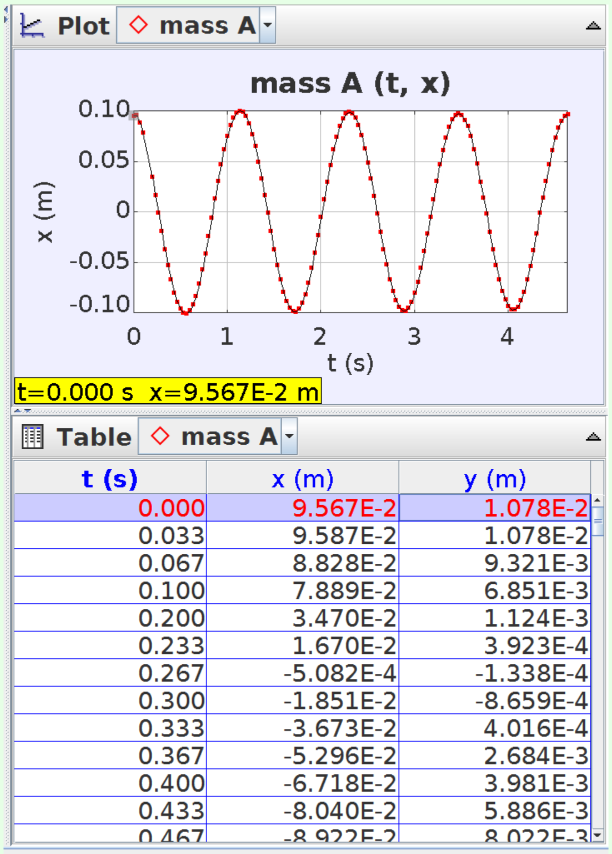
- Import video into Tracker (File / Import)
- A position reference system (x, y) is defined. In the image it is represented by the magenta axes.

- The reference length d_{ref} is inserted into the program.



- The position of the body is tracked in each frame of the movie of interest.

- As body position is tracked, Tracker offers both the $x(t)$ graph of the horizontal component of the position and the numeric data (see columns below). [Working in conditions of small oscillations we will neglect the deviations of the oscillating mass from the x direction.]

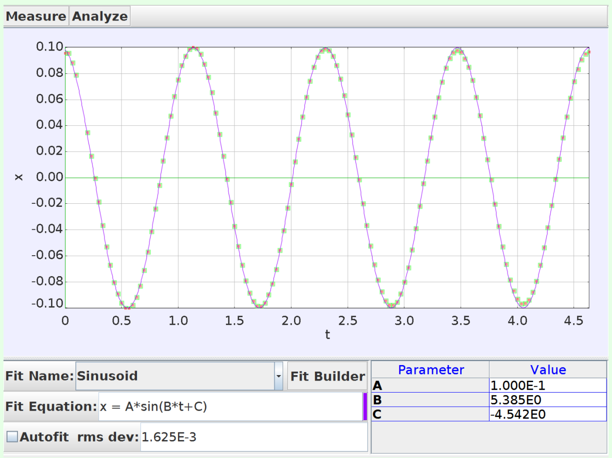


- By opening the "Data Tool (Analyze)" window (View / Data Tool (Analyze)) you have the possibility to fit the experimental points with a sinusoid of the type

$$x(t) = A\,sin(Bt + C)\,. \qquad (1)$$

Where A represents the amplitude, B the angular frequency ω, C the phase Phi of the harmonic motion.

- The fitting operation is done manually by changing the parameters A, B, C by hand (ie by entering the values directly) or with "clickable" arrows that allow a fine adjustment of their values. At the end of this operation we determine the value of ω, parameter B, which guarantees the best fit. In this example $\omega = 5.385\, s^{-1}$. The fitting result is shown in the figure below.



- From the value of ω we find the frequency f, to the period T and, finally, to the estimate of the absolute value of the acceleration of gravity g:

$$f = \frac{\omega}{2\pi}, \quad T = \frac{1}{f}, \quad g = \frac{4\pi^2 L}{T^2}. \tag{2}$$

All that remains is to evaluate the absolute uncertainty on the measurement of the acceleration of gravity.

Activity 1. Evaluation of uncertainties.

To evaluate the absolute uncertainty on g we need to use the usual calculation techniques of the error propagation. (on the website page you will find a short VIDEO which incorporates the technique of calculating uncertainty through differentiation).

- As a first approximation we can assign the instrumental uncertainty of our "stopwatch" to the period T. The temporal cadence is given by the acquisition frequency of our video camera (frame-rate or frame per second) which can be seen from the time data column, evaluating the distance between two successive instants. In the case, for example, the frame-rate is 30 Hz, so the instrumental uncertainty will be equal to $\delta t = \frac{1}{30} s = 0.03\, s$
- The uncertainty on L, measured with an extensible meter, can be considered equal to 0.5 cm. This choice guarantees us not to underestimate the errors due to a possible misalignment between the meter and the size in question.

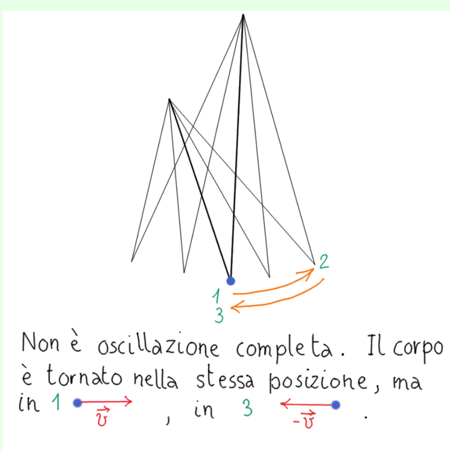
Activity 2. Repeated measures for determining the period of a pendulum and the acceleration of gravity.

The purpose of this experience is to determine the period of a simple pendulum starting from a series of repeated measures and using the obtained period to calculate the acceleration of gravity.

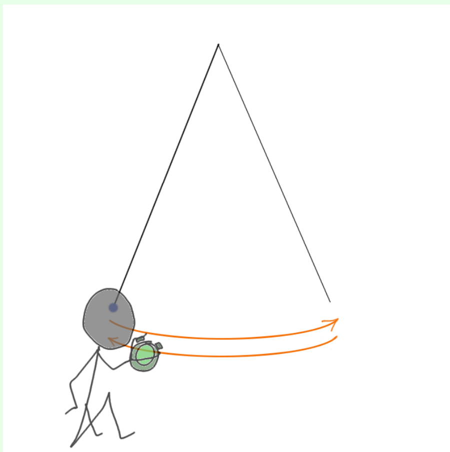
Let's start from the situation in which the pendulum has already been made following the indications of the section dedicated to the construction of the apparatus.

Activity 2. Operating procedure for measuring the period T.

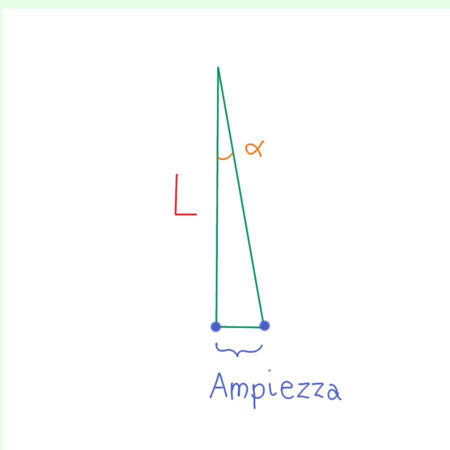
- Measure and note the pendulum length L.
- Establish and write down the uncertainty on L (we expect it to be greater than the sensitivity of the measuring instrument used, probably equal to 1 mm. Therefore, to avoid underestimating, we can assign the value δL to the uncertainty by 5 mm).
- Determine the number of oscillations whose duration will be measured (we recommend working with 5 oscillations). Recall that to have a complete swing it is not enough for the body to return to the previously occupied position, it is necessary to add the condition that also has the same direction and speed it previously had at that point.



- Position yourself beside the trajectory described by the pendulum at one of the two motion inversion points. Each time the pendulum returns at that point, an oscillation will have been completed. At that point, moreover, the pendulum will have zero speed so the measurement of the duration of the 5 oscillations will be more accurate.



- Swing the pendulum remembering to respect the condition of small oscillations (oscillation amplitude less than about 1/10 of the pendulum length L).



- Repeat the measurement of the duration of 5 oscillations 100 times and write down the 100 values $T_5^{(i)}$ measured (with i ranging from 1 to $N = 100$).

- Working on a spreadsheet (Excel, Libre Office, ...) evaluate the average $\overline{T}_5$:

$$\overline{T}_5 = \frac{1}{N} \sum_{i=1}^{N} T_5^{(i)} \tag{3}$$

- Evaluate the uncertainty associated with each individual measurement of the period of 5 oscillations. It will be given by the standard deviation of the 100 measurements collected:

$$\delta T_5 = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(T_5^{(i)} - \overline{T}_5)^2}$$

- Calculate the uncertainty on the mean $\overline{T}_5$:

$$\delta \overline{T}_5 = \frac{\delta T_5}{\sqrt{N}} \tag{4}$$

  This is a crucial point of the experiment, the relationship (4) represents the main reason why it is important to have a rich statistic (high value of N): the uncertainty on the average is reduced by a factor $\sqrt{N}$ compared to the error on the single measure.

- Evaluate the average of the pendulum period T, that is the duration of the single oscillation:

$$\overline{T} = \frac{\overline{T}_5}{5} \tag{5}$$

- Propagate uncertainty from $\overline{T}_5$ to $\overline{T}$:

$$\delta \overline{T} = \frac{\delta \overline{T}_5}{5} \tag{6}$$

- The outcome of the experiment is:

$$T = \overline{T} \pm \delta \overline{T}\,. \tag{7}$$

Once determined the pendulum period and its uncertaint, the acceleration of gravity can be estimated starting from the knowledge of T and L.

Activity 2. Acceleration of gravity calculation

- Use the relation that expresses the period as function of the pendulum length to calculate g starting from the period $\overline{T}$ and the length L derived from the experiment.

$$T^2 = \frac{4\pi^2}{g} L \,. \tag{8}$$

- Calculate the uncertainty on the value of g thus obtained, using the usual techniques for calculating the error propagation. (on the website page you will find a short VIDEO which incorporates the technique of calculating uncertainty through differentiation).

Activity 3. Study of the dependence of a pendulum's period on its length and determination of the acceleration of gravity.

The activity involves the collaboration of several students (more than or equal to 5). Each student will make a two-wire pendulum of different length and determine its period starting from repeated measurements lasting 4 complete oscillations (we recommend a number between 40 and 80 measurements). Once the individual's work is finished (for the experimental procedure see Activity 2) the collaborative phase of sharing and data analysis will begin.

By collecting the data of each individual student, the group will have a data set consisting of at least 5 pairs of values of the pendulum length and period:

$$(L_i, T_i) \tag{9}$$

where L_i is the pendulum length of the i-th student and T_i the corresponding period.

The purpose of this activity is to represent on a graph with the length L_i of the pendulums in the x-axis and the square of the periods, T_i^2 in the y-axis (Fig. 1). Then, with the least squares method, find the parameters a and b of the line that best fits the experimental data(Fig. 2):

$$y = ax + b \tag{10}$$

where $y = T^2$ and $x = L$. The gravity acceleration value must be deduced from the slope value of this line. Everything will be done in the light of simple pendulum theory, already knowing that the relationship between T^2 and L is a straight line through the origin with slope inversely proportional to g:

$$T^2 = \frac{4\pi^2}{g} L\,. \tag{11}$$

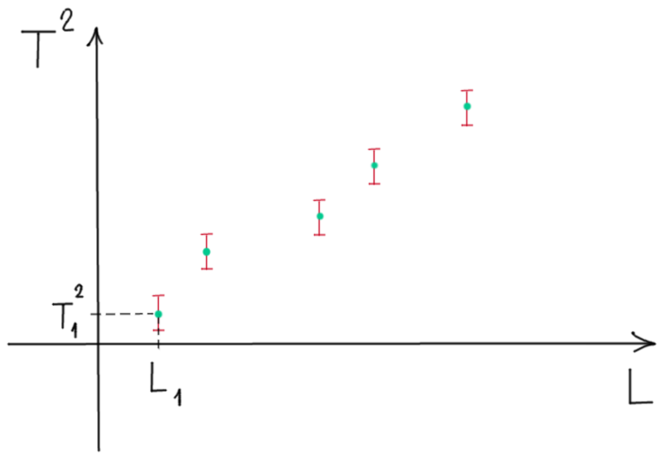


Figure 1: The plot of T^2 vs. L. Each point on the graph represents the result obtained by a member of the group.

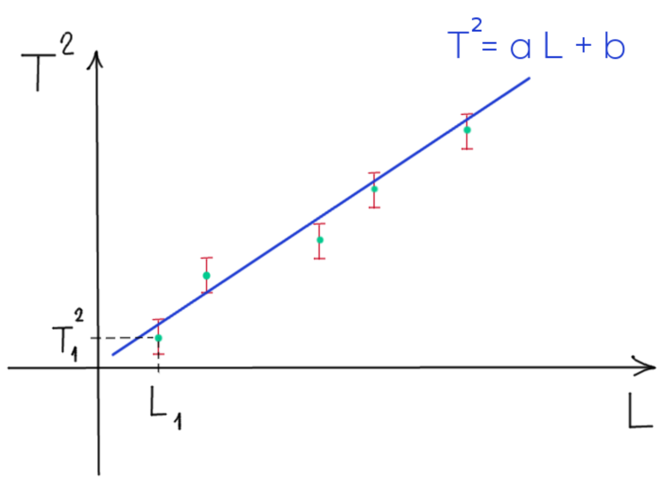


Figure 2: With the least squares method we will look for the values of the parameters a and b that best adapt the straight line to the experimental data

Activity 3. The least squares method: calculation of parameters a and b.

- The purpose is the determination of the straight line that best fits our experimental data (L_i, T_i^2). In particular, find the values of a, the angular coefficient, and b, the intercept, which define this straight line.
- The subscript i ranges from 1 to N, where N represents the number of data (in our example $N = 5$).
- From here on we will indicate the values of the independent variable L_i with the symbol x_i while the dependent variable T_i^2 will be indicated by the symbol y_i.
- Theory tells us that to determine the 2 parameters a and b, it is necessary to calculate 4 quantities:

$$\sum_{i=1}^{N} x_i \quad , \sum_{i=1}^{N} y_i \quad , \sum_{i=1}^{N} x_i^2 \quad , \sum_{i=1}^{N} x_i y_i$$

- A spreadsheet (Excel, Libre Office, ...) can help us in reaching our goal. First we can define two columns of data. One for the x_i, with the 5 pendulum lengths, the other for the y_i, containing the 5 corresponding period values squared.
- Now let's fill 2 new columns. In one, alongside the data columns, we will calculate x_i^2, in the next the product $x_i\, y_i$.
- At this point we should have 4 columns of 5 rows. In the first we will have the values of x_i, in the second the values y_i, in the third the x_i^2, in the fourth the $x_i\, y_i$.
- In order not to get confused we leave an empty row under these 4 columns and in the next row we calculate the sums of all the elements above. So, for example, under the x_i column we will have the $\sum_{i=1}^{N} x_i$.

- Now we just have to calculate the values of the parameters a and b of the straight line that will represent the linear link between our variables. We will have to use these formulas (all summations are to be understood with i ranging from 1 to N):

$$\Delta = N \sum x_i^2 - \left(\sum x_i\right)^2 \tag{12}$$

$$a = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta} \tag{13}$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta} \tag{14}$$

At this point we should have found the parameters a and b that define the straight line that best fits our data. We can represent it in superimposition to the graph of the 5 experimental points. In this way we will obtain a graph similar to that of the figure (2). By analyzing it, we will be able to immediately make two rough checks on the calculations carried out so far:

1. The line should adapt to our experimental points. If it passes away from them or it shows a slope in striking disagreement with the experimental data, it allow us to glimpse that probably some calculations will have got out of hand.

2. The theoretical model tells us that the linear relationship between T^2 and L is expressed by a straight line passing through the origin. We therefore expect a small value for the b parameter. Another alarm bell, so, will be any intercept values that are conspicuously different from zero.

Once these checks have been carried out, we can continue with the analysis. Now it is essential to calculate the uncertainty with which we have estimated the slope a and the intercept b of the line. The reason, for example, is that we will use the value of a to estimate g, we will therefore need the uncertainty of a to then propagate it on the uncertainty of the acceleration of gravity.

Activity 3. The least squares method: the uncertainty of a and b.

- The uncertainties on the parameters are calculated with these formulas:

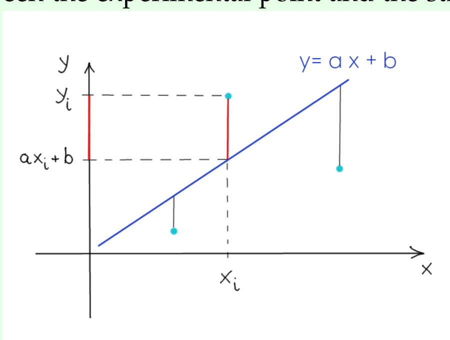
$$\delta a = \delta y \sqrt{\frac{N}{\Delta}} \quad , \quad \delta b = \delta y \sqrt{\frac{1}{\Delta}\sum x_i^2} \tag{15}$$

where δy represents the uncertainty on the experimental ordinates, that is, on the periods squared T_i^2. To understand, those indicated by the red error bars in the figure (1).

- The uncertainty δy can be calculated using this relationship

$$\delta y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(ax_i + b - y_i)^2}\,, \tag{16}$$

which expresses a sort of mean value of the quadratic difference between the straight line found and the experimental points. Looking at the figure below, in fact, it can be seen that the difference in the sum of the argument is, in modulus, the length of the red segment: the distance in the y direction between the experimental point and the straight line.



Once found the uncertainties on the parameters of the line that best fits the experimental points, we can refine our control over the results of the analysis. As mentioned above, we expect b to have a value close to zero. Now we are able to verify if the variability range of the parameter b, that is the interval $[b - \delta b, b + \delta b]$ contains zero inside it.

That said, let's move on to the final phase, that is to estimate the value of g starting from the slope a of the line.

Activity 3. Calculation of g and its uncertainty.

As already mentioned several times, from theory we know that the relationship between T^2 and L is expressed by the equation

$$T^2 = \frac{4\pi^2}{g} L \,. \tag{17}$$

Now, however, we are holding an estimate of the angular coefficient of this relationship: the parameter a. We can therefore write

$$a = \frac{4\pi^2}{g} \tag{18}$$

and from this equation we can derive the value of g.
We can proceed:

- We calculate g using the value of the parameter found with the least-squares method:

$$g = \frac{4\pi^2}{a}. \tag{19}$$

- Using the uncertainty of a (15) we evaluate the uncertainty δg on the value of g. For this purpose, the usual calculation technique for the propagation of the error is used. (on the website page you will find a short VIDEO which incorporates the technique of calculating uncertainty through differentiation).

- The activity ends by expressing the numerical value of g obtained with this procedure:

$$g = \frac{4\pi^2}{a} \pm \delta g. \tag{20}$$